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Phase transitions of large-N two-dimensional Yang–Mills and generalized Yang–Mills theories in the double scaling limit

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Abstract. The large-N behavior of Yang–Mills and generalized Yang–Mills theories in the double-scaling limit is investigated. By the double-scaling limit, it is meant that the area of the manifold on which the theory is defined, is itself a function of N. It is shown that phase transitions of different orders occur, depending on the functional dependence of the area on N. The finite-size scalings of the system are also investigated. Specifically, the dependence of the dominant representation on A, for large but finite N is determined.

1 Introduction

In recent years, two-dimensional Yang–Mills theory (YM_2) and generalized Yang–Mills theories (gYM_2's) have been extensively studied [1–16]. These are important integrable models that can shed light on some basic features of pure QCD in four dimensions. Also, there exists an equivalence between YM_2 and the string theory.

The starting point for making a correspondence between YM_2 and string theory is to study the large–N limit of YM_2. For example, as it is shown in [3, 6, 7], a gauge theory based on $SU(N)$ is split at large N into two copies of a chiral theory, which encapsulate the geometry of the string maps. The chiral theory associated to the Yang–Mills theory on a two–manifold $\mathcal{M}$ is a summation over maps from the two-dimensional world sheet (of arbitrary genus) to the manifold $\mathcal{M}$. This leads to a $1/N$ expansion for the partition function and observables that is convergent for all of the values of area $\times$ coupling constant on the target space $\mathcal{M}$, if the genus is one or greater.

Among the results obtained for YM_2 are the partition function and the expectation values of the Wilson loops of YM_2 in lattice-formualations [1, 17] and continuum-formulations [4, 11, 12]. The partition function and the expectation values of the Wilson loops of gYM_2's have been calculated in [9, 10]. All of these results are in terms of summations over the irreducible representations of the corresponding gauge group. In general, it is difficult to perform these summations explicitly and obtain more closed results. However, for large groups, these summations are dominated by some specific representations in some cases, and one can obtain closed-form expressions for those representations and the observables of the theory.

In [18], the large-N limit of the $U(N)$ YM_2 on a sphere was studied. There the dominant (or classical) representation was found and it was shown that the free energy of the $U(N)$ YM_2 on a sphere with the surface area $A < A_c = \pi^2$ has a logarithmic behavior [18]. In [13], the free energy was calculated for areas $A > \pi^2$, from which it was shown that the YM_2 on a sphere has a third-order phase transition at the critical area $A_c = \pi^2$, like the well-known Gross–Witten–Wadia phase transition for the lattice two-dimensional multicolour gauge theory [19, 20]. The phase structure of the large-N YM_2, generalized YM_2's, and non-local YM_2's on a sphere were further discussed in [14, 16, 21, 22]. The large-N limit of the partition function of YM_2 on orientable compact surfaces with boundaries was discussed in [23], and the large-N behavior of Wilson loops of YM_2 and gYM_2 on sphere was recently investigated in [24]. The critical behaviors of these quantities have been also studied.

In [25], $U(N)$-YM_2 theories were investigated, with the property that the area of the manifold on which the theory is defined depends on N. It has been shown there that for a specific parameterization of area (in terms of N), for which the area tends to infinity as N tends to infinity, there are finite-size effects at large-N. By this it is meant that, although one expects that for an infinite area the dominant representation be one for which the corresponding density is everywhere one, for specific values of the parameters the dominant representation is not that one. The difference between that representation and the expected one, however, vanishes in terms of the intensive quantities. That is, if one investigates the size of the rows of the Young tableau themselves, there is a difference. However, if this size-difference is scaled by N (divided by N) to obtain an

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intensive quantity, it vanishes at the thermodynamic limit. A similar argument holds also for the partition function too. There is a difference between the logarithm of the partition functions for the dominant representation and the expected representation. However, if one scales the logarithm of the partition function with N (i.e. if one divides it by N^2) to obtain intensive free energy, then this difference vanishes as N tends to infinity.

In this paper we want to study the large-N behavior of YM_2 and gYM_2's on a sphere, the area of which depends on N. In Sect. 2 some known results are reviewed, mainly to fix notation. In Sect. 3, the double scaling limit is introduced and various kinds of phase transitions are investigated. Here the emphasis is on the intensive quantities, which remain finite at the thermodynamic limit ($N \to \infty$). Any discontinuity in such quantities in the thermodynamic limit is called a phase transition. It is seen that such discontinuities are due to only two factors: a discontinuity in the behavior of the area in the thermodynamic limit, and a discontinuity in the behavior of the free-energy density in terms of the area (the Douglas–Kazakov phase transition). It is also shown that through the dependence of the area on a parameter, one can to some extent control the order of the transition. In Sect. 4, a simple example is introduced and the phases are explicitly studied for that example. Finally, in Sect. 5, the finite-size effects corresponding to the double-scaling limit are investigated and the result of [25] is generalized to the case of gYM_2.

2 The classical representation

Following [10,16], a generalized $U(N)$ Yang–Mills theory on a surface is characterized by a function Λ:

$$\Lambda(R) = \sum_{k=1}^{p} \frac{a_{\mathrm{k}}}{N^{k-1}} C_{\mathrm{k}}(R)\,, \tag{1}$$

where R denotes a representation of the group $\mathrm{U}(N)$, a_{k}'s are constants, and C_{k} are the Casimirs of the group defined through

$$C_{\mathrm{k}} = \sum_{i=1}^{N} \left[(n_i + N - i)^k - (N - i)^k\right]\,. \tag{2}$$

n_i's are nonincreasing integers characterizing the representation. It is assumed that p is even and $a_p > 0$. For simplicity, from now on it is further assumed that all a_{k}'s with odd k vanish. The partition function of such a theory on a sphere of surface area A is

$$\begin{aligned} Z(A) &= \sum_R d_R^2\, \exp[-A\Lambda(R)]\,, \\ &=: \sum_R e^{S(R)}\,, \end{aligned} \tag{3}$$

where d_R is the dimension of the representation R.

For large N, the summation on R is dominated by the so-called classical representation, which maximizes the product $d_R^2 \exp[-A\Lambda(R)]$, as was shown, for example, in [16]. To obtain this representation, it is convenient to introduce the new parameters

$$\begin{aligned} x &:= \frac{i}{N}\,, \\ n(x) &:= \frac{n_i}{N}\,, \\ h(x) &:= -n(x) - 1 + x\,, \end{aligned} \tag{4}$$

the density

$$\rho(h) := \frac{\mathrm{d}x}{\mathrm{d}h}\,, \tag{5}$$

and the function

$$G(z) := \sum_{k=0}^{p} a_{\mathrm{k}}(-z)^k\,. \tag{6}$$

Then ρ_{cl} (the density corresponding to the classical representation) is characterized by

$$\begin{aligned} &\frac{A}{2} G(z) - \int \mathrm{d}y\, \rho_{\mathrm{cl}}(y) \ln|z-y| = \text{const.}, \\ &\qquad \text{iff } \rho_{\mathrm{cl}}(z) \neq 1 \quad \text{and} \quad -a \le z \le a, \\ &\int_{-a}^{a} \mathrm{d}z \rho_{\mathrm{cl}}(z) = 1\,, \end{aligned} \tag{7}$$

where a is a positive number to be determined through the above conditions.

For areas smaller than a critical area (A_{c}), the density corresponding to the classical representation is everywhere less than one. For areas greater than A_{c}, there are places where ρ_{cl} is equal to one. Moreover, for $A \to \infty$, the density tends to

$$\rho_\infty(y) = \begin{cases} 1, & -\frac{1}{2} < y < \frac{1}{2} \\ 0, & \text{otherwise} \end{cases}\,. \tag{8}$$

The change of the behavior of ρ_{cl} at $A = A_{\mathrm{c}}$ induces a phase transition, which is of the third-order for the so-called typical theories, as discussed in [24], for example. That is, the free energy (density) of the system defined as

$$F := -\frac{1}{N^2}\, \log(Z) \tag{9}$$

exhibits a discontinuous behavior at this area, and the discontinuity is like $(A - A_{\mathrm{c}})^3$.

3 The double scaling limit

If the area A is itself a function of N, then the phase structure of the system may be different from that discussed above. To be more specific, let us take A to be a function of N and some parameter α, independent of N:

$$A = A(N, \alpha)\,. \tag{10}$$

The behavior of the system at $N\to\infty$, namely the density ρ corresponding to the classical representation and the free energy F, is determined through the value of A at $N\to\infty$. So, defining

$$\mathcal{A}(\alpha):=\lim_{N\to\infty}A(N,\alpha)\,, \tag{11}$$

it is seen that at the thermodynamic limit ($N\to\infty$)

$$\begin{aligned}F&=F(\mathcal{A})\,,\\ \rho_{\rm cl}(z)&=\rho(\mathcal{A},z)\,.\end{aligned} \tag{12}$$

The dependence of $\mathcal{A}$ on α determines the phase structure of the system. Let us assume that $\mathcal{A}$ is an increasing function of α. It may happen that $\mathcal{A}$ is a smooth function of α. Then the system exhibits a third-order phase transition if there exists an $\alpha_{\rm c}$ for which

$$\mathcal{A}(\alpha_{\rm c})=A_{\rm c}\,. \tag{13}$$

For α greater than $\alpha_{\rm c}$, the system is in the so-called strong phase, where $\rho_{\rm cl}$ is equal to one for some values of its argument, whereas for α less than $\alpha_{\rm c}$, the system is in the so-called weak phase, where $\rho_{\rm cl}$ is everywhere less than one.

$\mathcal{A}$ may be a discontinuous function of α at some $\alpha_{\rm c}$. In this case, it is obvious that the free energy is a discontinuous function of α, and we have a zeroth order transition at $\alpha_{\rm c}$. The discontinuity in $\mathcal{A}$ may be such that for α greater than $\alpha_{\rm c}$, $\mathcal{A}$ becomes infinite. In this case, the free energy is not only discontinuous, but exhibits an infinite jump at $\alpha_{\rm c}$, since the free energy becomes infinite at infinite area. An example illustrating this is

$$A(N,\alpha)=\frac{\alpha}{\alpha_1}\,A_{\rm c}+b\,N^{\alpha-\alpha_2}\,, \tag{14}$$

where b, α_1, and α_2 are positive constants, and $\alpha_1<\alpha_2$. One then has

$$\mathcal{A}(\alpha)=\begin{cases}\frac{\alpha}{\alpha_1}\,A_{\rm c}, & \alpha<\alpha_2\\ \infty, & \alpha>\alpha_2\end{cases}\,. \tag{15}$$

So, the system exhibits an infinite jump in the free energy at $\alpha=\alpha_2$. Moreover, for $\alpha<\alpha_2$, the area $\mathcal{A}$ is a smooth function of α, and is less than $A_{\rm c}$ for $\alpha<\alpha_1$ and greater than $A_{\rm c}$ for $\alpha>\alpha_1$. So, there is a third-order phase transition at $\alpha=\alpha_1$ as well. In terms of the density corresponding to the classical representation:

- $\alpha<\alpha_1$. In this case $\max(\rho_{\rm cl})<1$, that is, the system is in the weak phase.
- $\alpha_1<\alpha<\alpha_2$. In this case, $\max(\rho_{\rm cl})=1$, that is, the system is in the strong phase.
- $\alpha_2<\alpha$. In this case, $\rho_{\rm cl}=\rho_\infty$, that is, the density is everywhere equal to one.

Another example is

$$A(N,\alpha)=A_{\rm c}+\frac{b\,(\alpha-\alpha_{\rm c})^p\,N^{\alpha-\alpha_{\rm c}}+c\,(\alpha-\alpha_{\rm c})\,N^{\alpha_{\rm c}-\alpha}}{N^{\alpha-\alpha_{\rm c}}+N^{\alpha_{\rm c}-\alpha}}\,, \tag{16}$$

where b, c, and p are positive constants. It is seen that

$$\mathcal{A}(\alpha)=\begin{cases}\mathcal{A}_{\rm w}(\alpha):=A_{\rm c}+c\,(\alpha-\alpha_{\rm c}), & \alpha<\alpha_{\rm c}\\ \mathcal{A}_{\rm s}(\alpha):=A_{\rm c}+b\,(\alpha-\alpha_{\rm c})^p, & \alpha>\alpha_{\rm c}\end{cases}\,. \tag{17}$$

So, one obtains

$$F=\begin{cases}F_{\rm w}[A_{\rm c}+c\,(\alpha-\alpha_{\rm c})], & \alpha<\alpha_{\rm c}\\ F_{\rm s}\,[A_{\rm c}+b\,(\alpha-\alpha_{\rm c})^p]\,, & \alpha>\alpha_{\rm c}\end{cases}\,. \tag{18}$$

$F_{\rm w}$ and $F_{\rm s}$ are the free energies in the weak and strong phases, respectively. Noting that

$$F_{\rm s}(A)-F_{\rm w}(A)=q\,(A-A_{\rm c})^3+\dots\,, \tag{19}$$

where q is a positive constant, one obtains

$$\begin{aligned}F_{\rm s}-F_{\rm w}&=F_{\rm s}[\mathcal{A}_{\rm s}(\alpha)]-F_{\rm w}[\mathcal{A}_{\rm w}(\alpha)],\\ &=q[\mathcal{A}_{\rm s}(\alpha)-A_{\rm c}]^3+F_{\rm w}[\mathcal{A}_{\rm s}(\alpha)]-F_{\rm w}[\mathcal{A}_{\rm w}(\alpha)]+\dots,\\ &=qb^3(\alpha-\alpha_{\rm c})^{3p}+s\,[b(\alpha-\alpha_{\rm c})^p-c(\alpha-\alpha_{\rm c})]+\dots,\end{aligned} \tag{20}$$

where

$$s:=F'_{\rm w}(A_{\rm c})\,. \tag{21}$$

It is seen that if p is less than one, then there is a phase transition of the order p. If p is greater than one, then there is a transition of the order one. If p is equal to one, the phase transition is of the order one (for $b\neq c$), or three (for $b=c$).

4 A simple example

As a specific example, consider the following parameterization of the area A:

$$A(N,\alpha)=\beta+\left(\frac{N}{2}\right)^\alpha\,, \tag{22}$$

from which one obtains

$$\mathcal{A}=\begin{cases}\beta, & \alpha<0\\ \infty, & \alpha>0\end{cases}\,. \tag{23}$$

This is a simpler version of the first example in the previous section. It is simpler in the sense that for $\alpha<0$, $\mathcal{A}$ is constant and hence there is no third-order phase transition. There remains only a transition corresponding to an infinite jump in the free energy.

If $\alpha<0$, the model is in the weak phase for $\beta<A_{\rm c}=\pi^2$, and in the strong phase for $\beta>A_{\rm c}$. In both cases, the density function $\rho_{\rm cl}$ is not identical to one (for finite β), that is, there is a value y_0 where $\rho_{\rm cl}(y)<1$ if $|y|>|y_0|$. The precise value of y_0 depends on β. For $\alpha>0$, the density $\rho_{\rm cl}$ is identical to one, that is, $\rho_{\rm cl}$ is equal to ρ_∞.

Let us explicitly investigate the transition at $\alpha=0$. Because the area diverges for $\alpha>0$, it is seen that the free energy also diverges for $\alpha>0$. However, one can still compare the values of the free energy for two different representations. To do so, we follow the procedure introduced in [25]. For $\alpha > 0$, where $\rho_{\rm cl}$ is equal to ρ_∞, the dominant representation is the trivial representation and the corresponding parameters are

$$n_i = i, \quad -M \le i \le M\,. \tag{24}$$

The parameter i denotes the row of the Young tableau and has been taken between $-(N-1)/2$ and $(N-1)/2$, and

$$M := \frac{N-1}{2}\,. \tag{25}$$

For $\alpha < 0$, the parameters corresponding to $\rho_{\rm cl}$ are

$$n_i = \begin{cases} i, & |i| \le M-l \\ i+\tilde{n}_i, & M-l < |i| \le M \end{cases}, \tag{26}$$

where $\tilde{n}_i$ is a strictly increasing function of i. The parameters l and $\tilde{n}_i$ have to be determined by maximizing the action in the $\alpha > 0$ region. Denoting the actions corresponding to the representations (24) and (26) by S_0 and S, respectively, a calculation similar to that performed in [25] results in

$$S - S_0 = 4\,l^2\left[\left(\ln\frac{M}{l} - \frac{A}{4}\right)F_1 + F_2\right]. \tag{27}$$

Defining

$$\begin{aligned} u &:= \frac{i}{l}, \\ r(u) &:= \frac{\tilde{n}_i}{l}\,, \end{aligned} \tag{28}$$

for large N, the functions F_1 and F_2 can be written as

$$\begin{aligned} F_1 &= \int_0^1 \mathrm{d}u r(u), \\ F_2 &= \int_0^1 \mathrm{d}u\{r(u)[-\ln(u+r(u)) + 1 - \ln 2] \\ &\quad + u[\ln u - \ln(u+r(u))]\} \\ &\quad + \frac{1}{2}\int_0^1 \mathrm{d}u \int_0^1 \mathrm{d}v\, \ln\left[1 + \frac{r(u)-r(v)}{u-v}\right]. \end{aligned} \tag{29}$$

To find the configuration that maximizes $(S - S_0)$, one puts equal to zero the variations of $(S - S_0)$ with respect to l and r. Similarly to [25], these equations result in

$$\ln\left(\frac{M}{l}\right) - \frac{1}{4}M^\alpha + \frac{F_2}{F_1} - \frac{\beta}{4} - \frac{1}{2} = 0\,, \tag{30}$$

and

$$\begin{aligned} F_1 &= \frac{1}{4}, \\ \frac{F_2}{F_1} &= \frac{1}{2}\,. \end{aligned} \tag{31}$$

Note that our F_2 differs from one used in [25] by $-\beta F_1/4$. Using (30) and (31), one obtains

$$\begin{aligned} l &= M e^{-\frac{1}{4}(M^\alpha + \beta)}, \\ &= M e^{-\frac{A}{4}}\,. \end{aligned} \tag{32}$$

Also, using (27), (30), and (31), one finds

$$\begin{aligned} S - S_0 &= \frac{l^2}{2}, \\ &= \frac{M^2}{2} e^{-\frac{1}{2}(M^\alpha+\beta)}, \\ &= \frac{M^2}{2} e^{-\frac{A}{2}}. \end{aligned} \tag{33}$$

If the partition function is dominated by one representation, (9) can be rewritten as

$$F = -\frac{S}{N^2}\,, \tag{34}$$

from which,

$$\begin{aligned} F - F_0 &= -\frac{1}{8} e^{-\frac{1}{4}(M^\alpha+\beta)}\,, \\ &= -\frac{1}{8} e^{-\frac{A}{2}}\,, \end{aligned} \tag{35}$$

or in the large-N limit,

$$F - F_0 = \begin{cases} -\frac{1}{8} e^{-\frac{\beta}{4}}, & \alpha < 0 \\ 0, & \alpha > 0 \end{cases}. \tag{36}$$

This shows explicitly that for $\alpha > 0$, the dominant representation is the one corresponding to the density ρ_∞.

Calculations similar to this can be performed for the $G(z) = z^k$- gYM$_2$ model, with even k. To do so, one simply has to change in S

$$-\frac{A}{2M}\sum_{i=1}^{M} n_i^2 \to -\frac{A}{(2M)^{k-1}}\sum_{i=1}^{M} n_i^k\,. \tag{37}$$

One also has

$$n_i^k - i^k = k i^{k-1}\tilde{n}_i + \dots, \tag{38}$$

where only the leading term has been kept.

Using these formulations, it is seen that for large N, the only change in the results comes through

$$A \to A' := \frac{k}{2^{k-1}} A\,. \tag{39}$$

It is then easy to see that (36) is changed to

$$F - F_0 = \begin{cases} -\frac{1}{8} e^{-\frac{k\beta}{2^{k+1}}}, & \alpha < 0 \\ 0, & \alpha > 0 \end{cases}. \tag{40}$$

Again we have a transition at $\alpha = 0$.

5 Finite-size effects

Suppose that the parameterization of A in terms of α is such that for $\alpha > \alpha_{\rm c}$, the area A is infinite. It is then obvious that for $\alpha > \alpha_{\rm c}$, and at the thermodynamic limit $N \to \infty$, the dominant representation is that corresponding to ρ_∞. The question arises: if N is large but not infinite, is it still the representation that dominates the partition function? The question may be rephrased as follows. Take another representation, for which the density ρ is not identical to one, and compare $S(R)$ in (3) for these two representations.

This has been done in [25] for YM_2 and for the parameterization

$$A = \alpha \log N + \beta \,. \tag{41}$$

The result obtained there is that $\rho_{\rm cl}$ (the density corresponding to the dominant representation) is equal to one, except for in a narrow region around $(\pm 1/2)$, that is,

$$\rho_{\rm cl}(z) \begin{cases} = 1, & |z| \le \frac{1}{2} - \epsilon \\ < 1, & |z| > \frac{1}{2} - \epsilon \end{cases} , \tag{42}$$

where

$$\epsilon \sim N^{-\alpha/4} \,. \tag{43}$$

In terms of the parameters n_i characterizing the representation ((25) and (26)), as calculated in [25], it turns out that for the dominant representation R one has

$$\begin{aligned} l &= e^{-\frac{\beta}{4}} M^{1-\frac{\alpha}{4}} \,, \\ &= M e^{-\frac{A}{4}} \,, \end{aligned} \tag{44}$$

and

$$\begin{aligned} S - S_0 &= \frac{l^2}{2} \,, \\ &= \frac{1}{2} e^{-\frac{\beta}{2}} M^{2-\frac{\alpha}{2}} \,, \\ &= \frac{M^2}{2} e^{-\frac{A}{2}} \,. \end{aligned} \tag{45}$$

It is clear that for $\alpha < 4$, both l and $(S - S_0)$ diverge as N (or equivalently M) tends to infinity; whereas for $\alpha > 4$, both l and $(S - S_0)$ tend to zero at the thermodynamic limit. However, if one considers quantities properly scaled by N (so that they do not diverge at the thermodynamic limit), one should investigate the behaviors of (l/N) and F (defined through (9)) rather than those of l and S. If the partition function is dominated by a representation, then (9) is rewritten like (34). Equations (44) and (45) are then transformed to

$$\begin{aligned} \frac{l}{N} &= \frac{1}{2} e^{-\frac{\beta}{4}} M^{-\frac{\alpha}{4}}, \\ &= \frac{1}{2} e^{-\frac{A}{4}} \,, \end{aligned} \tag{46}$$

and

$$\begin{aligned} F - F_0 &= -\frac{1}{8} e^{-\frac{\beta}{2}} M^{-\frac{\alpha}{2}} \,, \\ &= -\frac{1}{8} e^{-\frac{A}{2}}. \end{aligned} \tag{47}$$

It is obvious that in terms of these quantities, $\alpha = 4$ is no specific point. That is, (l/N) and $(F - F_0)$ both tend to zero as N tends to infinity, as long as α is positive. This is expected, as for positive α, the area A is infinite at the thermodynamic limit, and for infinite area, the dominant representation is that corresponding to ρ_∞.

Calculations similar to this can be performed for $z^k-{\rm gYM}_2$ (with even k). To do so, one simply has to apply the changes (37) and (38).

Using these formulations, it is seen that for large N, the only change in the results of [25] comes through (39). It is then obvious that defining

$$\begin{aligned} \alpha' &:= \frac{k}{2^{k-1}} \alpha \,, \\ \beta' &:= \frac{k}{2^{k-1}} \beta \,, \end{aligned} \tag{48}$$

for l and $(S - S_0)$ (or (l/N) and $(F - F_0)$) one obtains results very similar to (44) and (45) (or (46) and (47)), but with α and β replaced by α' and β'. So, at the thermodynamic limit, for $\alpha < (2^{k+1}/k)$ both l and $(S - S_0)$ diverge, while for $\alpha > (2^{k+1}/k)$ both l and $(S - S_0)$ tend to zero. Also, at the thermodynamic limit both (l/N) and $(F - F_0)$ tend to zero for any positive α, as expected from the behavior of the density corresponding to the dominant representation for large areas.

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